Elliptic Flow in Central Collisions of Deformed Nuclei

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Non-trivial geometrical effects in relativistic central collisions of deformed nuclei are studied using a simple version of optical Glauber model. For very small impact parameters large centrality and eccentricity fluctuations are observed. In very high multiplicity collisions of oblate nuclei (e.g. Au^{197} and Cu) a significant fraction of events with elliptic flow strength v_2 dependent on oblateness β_2 is predicted.

I.   INTRODUCTION.

In relativistic heavy-ion collisions a simple relation between *centrality* of collisions $c(N)$ and average participant *eccentricity* of interacting nucleons $< \epsilon_p >$ is usually expected to be valid. According to rather general assumptions [1] centrality $c(N)$ is directly related to impact parameter b as $c(N) \approx \pi b^2/\sigma_{inel}$. At the same time the participant eccentricity

$$\epsilon_p(b) = \sum_i (x_i^2 - y_i^2)/\sum_i (x_i^2 + y_i^2) \qquad (1)$$

(where x_i and y_i are positions of interacting nucleons in transversal plane) is also singly dependent (within fluctuations) on the size of impact parameter b. Thus measured elliptic flow strength v_2 is expected to rise [2, 3] from very small values in most central collisions proportionaly to increasing values of average eccentricity $v_2 \approx k \cdot < \epsilon_p(b) >$.

However these simple relations are not valid for collisions of deformed nuclei. The orientation of a deformed nucleus (e.g. relative to beam axis) has a direct influence on centrality and eccentricity of collisions at a given fixed value of impact parameter b. This is slightly worrying since the elliptic flow strength measured in collisions of nuclei having non-zero deformation β_2 (e.g. Au, Cu) has been interpreted [4] assuming these nuclei to be spherical.

In the following sections we present a preliminary study of non-trivial variations of initial eccentricity at given collision centralities in relativistic collisions of deformed nuclei.
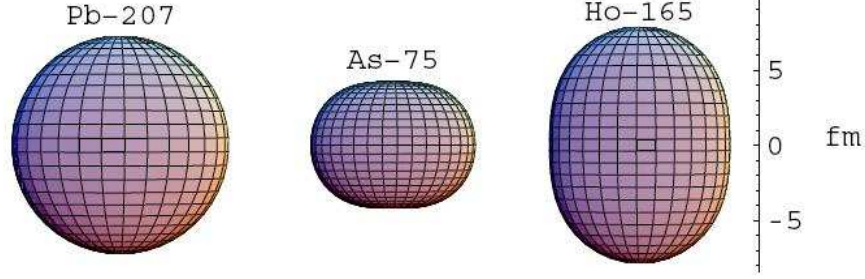
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Fig.1: Predicted shapes of Pb^{207}, As^{75} ($\beta_2 = -0.25$) and Ho^{165} ($\beta_2 = 0.3$) nuclei [9].

II.   GEOMETRICAL EFFECTS IN COLLISIONS OF DEFORMED NUCLEI

Although effects of nuclear deformation have been carefully studied in fusion reactions at coulomb barrier [5] the role of nuclear deformation in relativistic heavy ion collisions have been considered scarcely so far. It had been pointed out [6] that highest energy densities of QCD matter could be created in central collisions of longitudinaly oriented heavy deformed nuclei and elliptic flow in such collisions had been discussed. However since the heaviest nuclei suitable for relativistic collisions experiments are prolate [9] the elliptic flow in central collisions of oblate nuclei have escaped the attention of heavy ion community so far.

In this contribution we point out that oblate ground-state nuclear deformation of stable nuclei can have a significant influence on measured elliptic flow values in central collisions since it directly influences fluctuations and average values of initial eccentricity ϵ_p at given collision centralities.

Let us consider angle θ between the beam direction and the main axis of a deformed nucleus (e.g. Ho-165). For $\theta \approx 0$ nucleus is longitudilay polarized and for $\theta \approx \pi/2$ we have transversal polarization. Even for very small impact parameters $b \approx 0$ (central collisions geometrically) the orientation of a deformed nucleus relative to beam axis influences directly participant eccentricity

$$\epsilon_p(\theta, b) = \frac{\sum^i x_i^2 - y_i^2}{\sum^i x_i^2 + y_i^2} \tag{2}$$

and thus final strength of the elliptic flow observed. Also *centrality* of collision depends on the orientation of a deformed nucleus at given impact parameter. This happens not only due to changes in the number of participating nucleons (a simple overlap effect) but also due to that fraction of secondary particles multiplicity which is proportional to total number of binary nucleon-nucleon collisions [7].

Thus geometric relation between centrality and impact parameter [1] is not valid for

collisions of deformed nuclei and initial eccentricity ϵ_p is not simply related to collision *centrality* as it is usually assumed. This can in principle modify interpretations drawn from the eliptic flow strength v_2 at given centrality $c(N)$ in Au+Au collisions at RHIC.

III.   A SIMPLE OPTICAL GLAUBER SIMULATION

In order to test quantitatively the effects of nuclear deformation on centrality and eccentricity in heavy ion collisions a simple Optical Galuber Model (OGM) calculation for deformed nuclei had been performed. Here is a very short description of the simulation:

In the first step a deformed Woods-Saxon [8] density

$$\rho(\vec{r}) = \frac{\rho_o}{1 + e^{(r - R_0(1 + \beta_2 Y_{20} + \beta_4 Y_{40}))/a}} \tag{3}$$

with deformation parameters β_2, β_4 taken from [9] for various nuclei (Au,Cu,Ho,Pb,Si,Ca) had been used to generate transversal projections $\rho^T(x,y) = \int \rho(\vec{r}) dz$ of nucleon density. Binning of $\rho^T(x_i, y_j)$ histograms was $0.1 fm$ and longitudinal integration of deformed Woods-Saxon density $\rho(\vec{r})$ was performed in $0.05 fm$ steps. For deformed nuclei separate projections $\rho^T[\theta, \phi]$ had been generated at fixed values of angle θ and azimuthal angle ϕ.
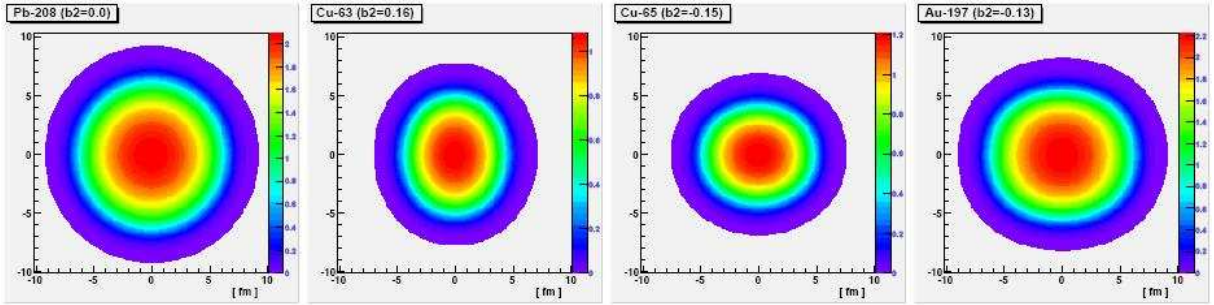

Fig.2: Projected nucleon densities $\rho^T(x,y)$ for Pb^{208}, $^\uparrow Cu^{63}$, $^\uparrow Cu^{65}$, $^\uparrow Au^{197}$ (β_2 from [9]).

In order to simulate also non-central collisions transversal projections $\rho^T(x,y)$ had been shifted for selected values of impact parameter $\vec{b} = (b_x, b_y)$ and total number of nucleon-nucleon collisions $N_{coll.}$ had been evaluated as

$$N_{coll} = \sum_{i,j} N_{coll}(x_i, y_j) = \sum_{i,j} \sigma_{NN}^{inel} \cdot \rho_1^T(x_i - b_x/2, y_j - b_y/2) * \rho_2^T(x_i + b_x/2, y_j + b_y/2). \tag{4}$$

The cross section σ_{NN}^{inel} was fixed at $42 mb$.

Using the geometrical information stored in histograms $\rho_1^T(x_i, y_i)$ and $\rho_2^T(x_i, y_i)$ transversal projected baryon density $\rho_B(x, y)$ and number of participants N_{part} were simply evaluated

and found to be in good agreement[1] with results of Monte-Carlo Glauber model simulation [10] as shown in Figs.9-10 (Appendix).

Eccentricity $\tilde{\epsilon}_{NN}$ of the interacting volume had been evaluated as

$$\tilde{\epsilon}_{NN} = \frac{\sum_{i,j} N_{coll}(x_i, y_j) \cdot (x_i^2 - y_j^2)}{\sum_{i,j} N_{coll}(x_i, y_j) \cdot (x_i^2 + y_j^2)} \tag{5}$$

in CMS coordinates of the overlaping zone. Numerical values of eccentricity $\tilde{\epsilon}_{NN}$ were found to be in reasonable agreement with Au+Au eccentricities evaluated in [3] (see Fig.11 in Appendix). Selected results of these calculations for Au,Cu,Ho,Pb,Si nuclei are presented in the following sections.

IV.   COLLISIONS OF SPHERICAL + DEFORMED NUCLEI

For the sake of simplicity let us start with polarized collisions of a deformed nucleus e.g. a prolate Ho-165 ($\beta_2 \approx 0.3$) with spherical projectile Pb-207.
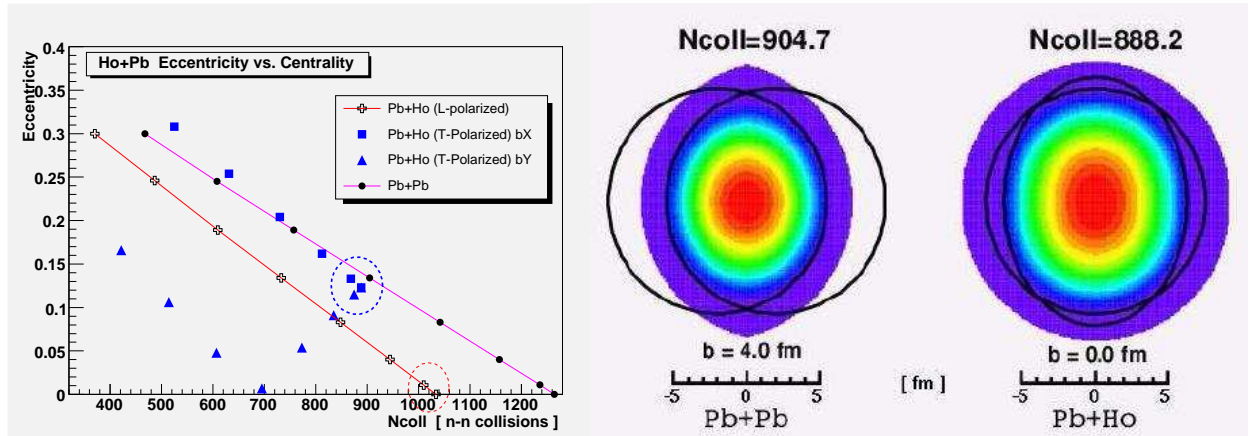

Fig.3: Eccentricity[n-n collisions] plot for Pb+Pb and polarized $^\uparrow$Ho+Pb systems.

If the spin$^{(7/2)}$ of Ho-165 nucleus is orthogonal to beam axis (T-polarized) the participant eccentricity ϵ_p will be non-zero even for $b = 0$ its value roughly corresponding to non-central Pb+Pb collisions at $b = 4$ fm (see Fig.3). Increasing impact parameter values (b_x, b_y) in directions parallel (b_y) and orthogonal (b_x) to Ho^{165} spin have different influence on the eccentricity. For increasing $(b_x, 0)$ the eccentricity further increases while for increasing

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[1] Spherical shape for Au-197 with $R = 6.38$fm and diffusivity $a = 0.53$fm was used for this comparison.

$(0, b_y)$ the eccentricity first decreases to zero at $b_y \approx 4$ fm and then it increases as it can be seen from Fig.3 (data points are evaluated for impact parameters increasing in steps 1fm).

For longitudinal polarization of holmium nuclei (relative to beam direction) the initial eccentricity tends to zero in the most central collisions. This happens because eccentricity fluctuations [11] are not taken into account in our simple optical Glauber calculation. It is also visible from Fig.3 that central collisions of L-polarized Ho-165 with Pb have significantly higher multiplicity compared to central T-polarized Ho+Pb collisions (with $b = 0$). Thus the highest multiplicity collisions of prolate nuclei correspond to longitudinal orientations of deformed nucleus due to the highest number of nucleon-nucleon interactions in this case.

This means that in order to study the elliptic flow in central collisions of transversaly oriented prolate nuclei with spherical projectiles one needs to select events with lower measured multiplicity then maximal. In such sample of events non-central collisions of longitudinaly oriented prolate nuclei will get mixed together with central collisions ($b \approx 0$) of transversaly oriented prolate nuclei of similar multiplicity (see Fig.3). One can try to distinguish these two types of events via measurement of spectator energy in ZDC calorimeters and to perform target (or beam) polarization-dependent experiments (see [12] for polarized Ho-165 target).
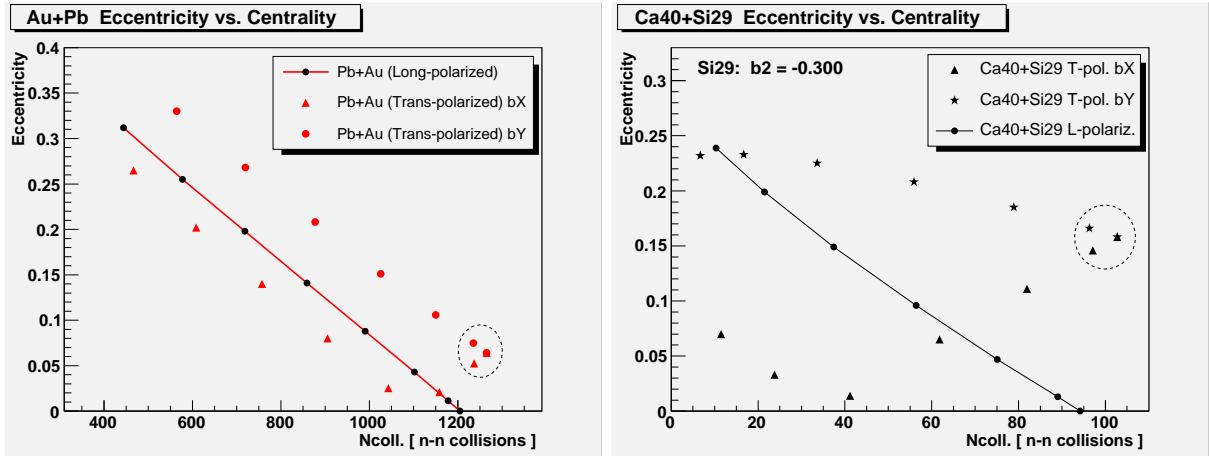

Fig.4: Eccentricity plots for polarized $^{\uparrow}Au^{197}+Pb^{207}$ and $^{\uparrow}Si^{29}+Ca^{40}$ collisions.

Fortunately the existence of nuclei with oblate ($\beta_2 < 0$) deformation (Au,I,Ga,As,Si,Al) provides us with another possibility in this direction. For oblate nuclei very high multiplicity collisions correspond to transversal polarization of nuclear spin relative to the beam direction due to higher number of nucleon-nucleon collisions in this case. Thus significant fraction of very central collisions of oblate nuclei should exhibit non-zero elliptic flow v_2 values

proportional to deformation parameter $\beta_2 < 0$. In Fig.4 we show results of our calculations for $Au^{197}+Pb^{207}$ and $Si^{29}+Ca^{40}$ collisions. A small difference in the number[2] of nucleon-nucleon interactions in the most central collisions with longitudial and transversal spin polarizations is obvious. In the next section we show that the influence of nuclear ground-state deformation on the eccentricity fluctuations in Au+Au and Cu+Cu collisions (studied at RHIC) is expected to be more significant.

V.   COLLISIONS OF TWO DEFORMED NUCLEI

In collisions of two deformed nuclei various geometrical configurations are possible. Let us denote θ_1 and θ_2 to be angles of deformed nuclei main axes relative to beam direction and ϕ_1 and ϕ_2 to be main axes azimuthal angles in LAB. For unpolarized beams (targets) azimuthal angles are independent and nuclei collide at random orientations of ϕ_1, ϕ_2. A trivial consideration shows that probability of having a collision with deformed nuclei axes oriented at *relative* azimuthal angle $0 < \Delta\phi < \pi/2$ is constant: $P(\Delta\phi) = const.$

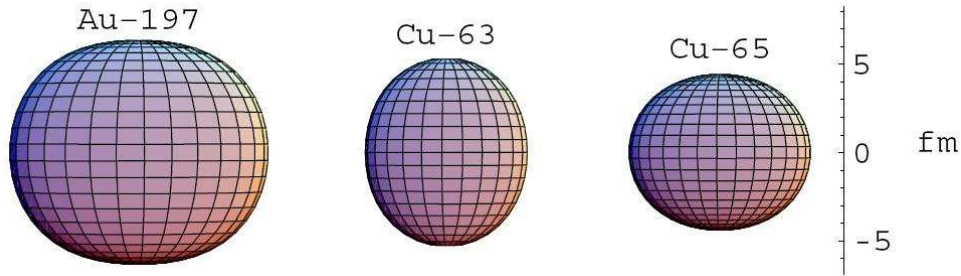**Fig.5:** Shape of Au^{197}, Cu^{63}, Cu^{65} with deformation $\beta_2 =$[-0.13/+0.16/-0.15] taken from [9].

Polar angles θ_1, θ_2 are also mutually independent but not random and probability $P(\theta_1, \theta_2)$ of a collision of two nuclei oriented at angles $0 < \theta_1 < \pi$ and $0 < \theta_2 < \pi$ is

$$P(\theta_1, \theta_2) = \sin(\theta_1)\sin(\theta_2)/4 \qquad (6)$$

which says that longitudinal-longitudinal oriented (Long-Long) collisions of deformed nuclei are rather rare while transversal (Trans-Trans) collisions ($\theta_1 \approx \theta_2 \approx \pi/2$) are more frequent.

One should keep in mind that this applies to collisions at any value of geometrical impact parameter occuring with probability $P(b < b_{max}) = c \cdot b_{max}^2$. Therefore central ($b \approx 0$)

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[2] One should always keep in mind the existence of fluctuations in the number of n-n collisions.

Long-Long collisions are very rare! Nevetherless, just such collisions of *prolate* nuclei can be selected [6] from the sample of events according to the centrality (high-multiplicity) criterion and analyzed.
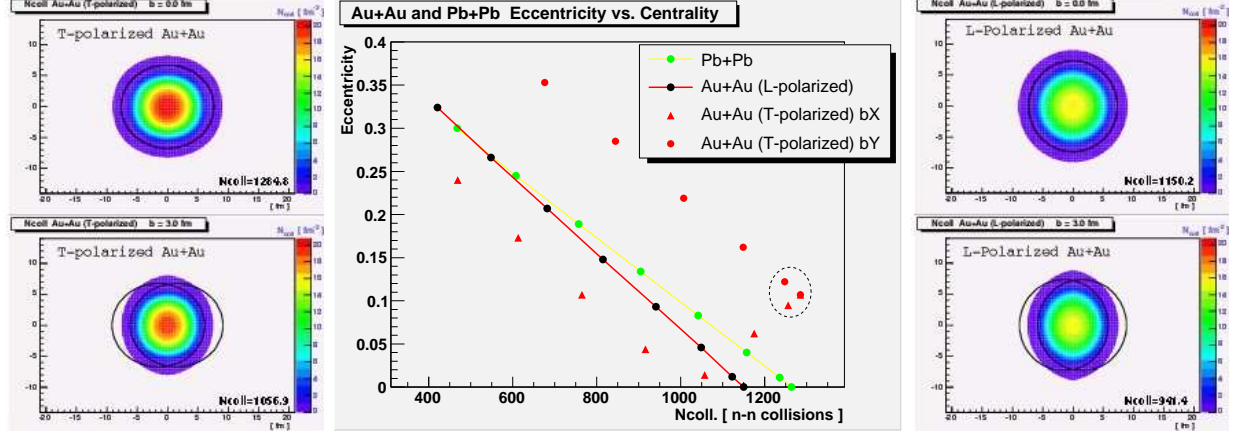

Fig.6: Densities $N_{coll}(x,y)/\text{fm}^2$ for $\text{Au}^\uparrow+\text{Au}^\uparrow$ at b_x=0fm/3fm and Eccentricity[Ncoll] plot.

As can be seen from Fig.6 Trans-Trans collisions of *oblate* nuclei with parallel orientation of nuclear spins ($\Delta\phi = 0$) are the highest multiplicity collisions for nuclei with oblate geometry. They can be localized in the sample of events based on the collision centrality and elliptic flow v_2 which is expected to be increasing with oblateness strength -β_2.
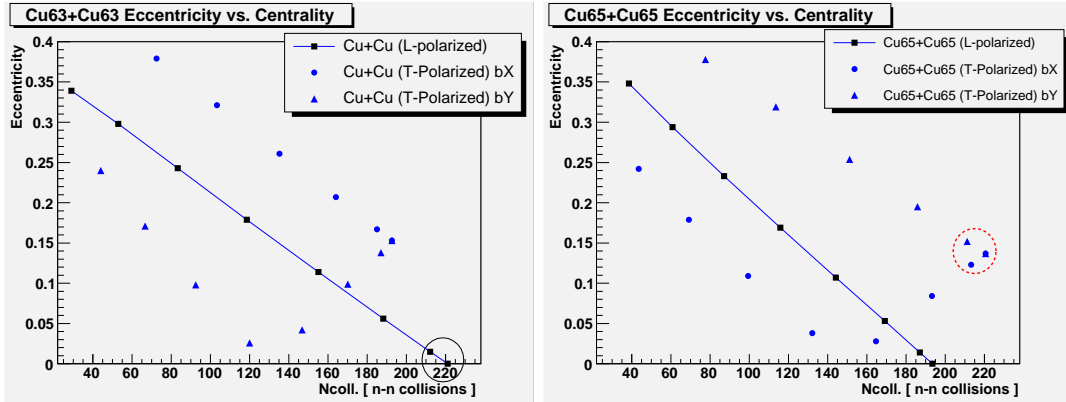

Fig.7: Ecc[Ncoll] plot for $\text{Cu}^{63}+\text{Cu}^{63}$ and $\text{Cu}^{65}+\text{Cu}^{65}$ assuming $\beta_2 = 0.16$ and -0.15 [9].

If stable copper isotopes are deformed as predicted in [9] then most central collisions of Cu^{63} should exhibit significantly different elliptic flow strength v_2 compared to most central collisions of $\text{Cu}^{65}+\text{Cu}^{65}$ (predicted $\beta_2 = -0.15$). This means that elliptic flow measured in most central collisions of a given isotope can be used to determine β_2 sign of its ground-state deformation. This might hold also for spin-zero nuclei e.g. $\text{Si}^{28}_{0^+}$ (predicted $\beta_2 = -0.48$ [9]).

Situation is however slightly more complex than indicated in Figs. 6 and 7. In the case of Trans-Trans collisions ($\theta_1 \approx \theta_2 \approx \pi/2$) of deformed nuclei there is a freedom in relative azimuthal orientation $\Delta\phi$ of nuclear spins. For $\Delta\phi \approx \pi/2$ (orthogonal spins) elliptic flow should tend zero and for $\Delta\phi \approx 0$ (parallel spins) participant eccentricity and thus also the observed elliptic flow strength v_2 should be maximal (whatever the sign of β_2 is).
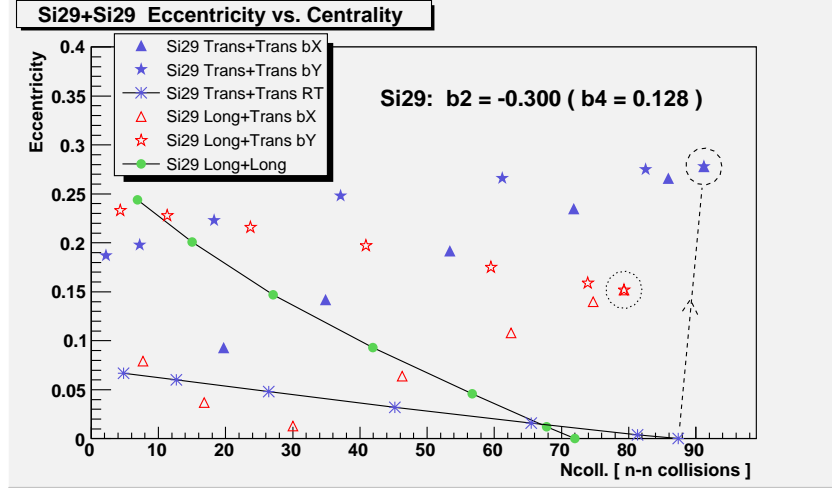

Fig.8: Eccentricity[Ncoll] plot for $^{\uparrow}Si^{29}+^{\uparrow}Si^{29}$ assuming β_2=-0.3 and β_4=0.13 [9].

This is clearly visible from our calculations in Figure 8, where dashed arrow indicates increase of eccentricity for transversally polarized $\theta_1 = \theta_2 = \pi/2$ collisions of $Si^{29}+Si^{29}$ when azimuthal angle difference of spins changes from $\Delta\phi = \pi/2$ (denoted as Trans-Trans RT) to parallel spins orientation $\Delta\phi = 0$. Therefore in geometrically most central collisions (zero impact parameters b) the participant eccentricity depends on polar angles θ_1, θ_2 and on relative azimuthal angle difference $\Delta\phi$:

$$\epsilon_p[b \approx 0] = \epsilon_p(\theta_1, \theta_2, \Delta\phi) \tag{7}$$

For random relative azimuthal angles $\Delta\phi$ a sample of very high multiplicity events will contain mixture of collisions with different $\Delta\phi$ and various eccentricities. This applies also to Au+Au collisions studied at RHIC. Collisions of longitudinaly+transversaly polarized Si^{29} have lower maximal number of binary collisions $N_{coll} \approx 80$ (at b=0 fm) with eccentricity $\tilde{\epsilon}_{NN} \approx 0.15$ (see Fig.8). Data points in all our Eccentricity[Ncoll.] plots correspond to impact parameters b_x and b_y increasing in steps 0,1,2,3,4,5,6 fm. Calculated number of binary collsions $N_{coll.}$ decreases with increasing impact parameters $(b_x, 0)$ and $(0, b_y)$.

As a matter of completeness we mention here also the possibility to collide prolate nuclei ($\beta_2 > 0$) with oblate nuclei ($\beta_2 < 0$) such as Ho^{165}+As^{75} or Cf^{251}+Au^{197}. In this case collisions with highest multiplicities will be Long-Trans polarized and they should exhibit elliptic flow strength $< v_2 >$ rising with β_2 deformation of the prolate nucleus.

VI. DISCUSSION OF RESULTS

It has been shown that in collisions of transversaly polarized prolate nuclei (e.g. Ho-165) with spherical projectiles (e.g. Pb-207) a significat elliptic flow will be generated even for zero impact parameters. Such collisions however will have similar multiplicity (centrality) as non-central collisions with longitudinaly polarized prolate nuclei. Careful polarization-dependent studies and precise measurement of spectator energy can distinguish different orientations of prolate nuclei relative to beam axis in such collisions. Highest-multiplicity collisions of prolate nuclei (such as Ho-165) correspond to longitudial orientations of prolate nuclei. Elliptic flow strength v_2 will thus tend to zero (within fluctuations) in such collisions.

For oblate nuclei the situation is different. Highest multiplicity collisions of oblate nucleus with a spherical projectile correspond to transversal polarization of oblate nucleus relative to beam axis. Additionally if both nuclei colliding are prolate highest multiplicity collisions will tend to be collisions of transversally polarized nuclei with parallel azimuthal orientation of nuclear deformation axes. Average value of elliptic flow strength v_2 measured in very high multiplicity (VHM) collisions of oblate nuclei will thus contain two contributions:

$$< v_2 >^{VHM} = < v_2 >_{fluct} + < v_2 >_{\beta_2} \tag{8}$$

The first contribution $< v_2 >_{fluct.}$ corresponds to eccentricity fluctuations which are present also in the highest multiplicity collisions of spherical nuclei. The second contribution is proportional to geometrical deformation of oblate ($\beta_2 < 0$) nucleus. Quantities $< v_2 >_{fluct}$ and $< v_2 >_{\beta_2}$ can in principle be disentangled if elliptic flow for highest multiplicity collisions of spherical nuclei with similar multiplicity is measured $< v_2 >^{VHM} = < v_2 >_{fluct}$. This approach can be attempted for two isotopes of a given element (e.g. Si^{29} and Si^{30}) or for two nuclei (spherical+oblate) with similar number of nucleons e.g. Pb and Au (see Fig.6).

One should keep in mind still that $< v_2 >_{\beta_2}$ contains averaged values of $v_2(\Delta\phi)$ due to

almost[3] random relative azimuthal orientations $\Delta\phi$ of nuclear spins in such highest multiplicity events (for unpolarized beams and targets).

VII.   CONCLUSIONS

A simple simulation based on optical limit approximation of the Glauber model [13] shows that very high multiplicity collisions of oblate nuclei at high energies should exhibit non-zero elliptic flow strength dependent on β_2 deformation parameter of the oblate nucleus. This provides us with the possibility to study the elliptic flow phenomenon in the most central ultra-relativistic collisions of oblate nuclei. The equation of state of QCD matter can thus be investigated in the highest multiplicity collisions. The energy dependence [14] of elliptic flow strength v_2 in these very high multiplicity (VHM) central collisions of oblate nuclei can reveal changes in the equation state of hadronic matter (e.g. kink-like signatures [15, 16]) due to the expected phase transition of QCD matter.

For a precise investigation to what extent the oblateness of Au^{197} nucleus influences interpretations drawn from elliptic flow measurements at RHIC [4] a full MC simulation based on Glauber model taking into account ground-state deformation of Au^{197} nucleus together with all possible orientations $\theta_1, \theta_2, \phi_1, \phi_2$ of colliding nuclei is probably necessary. A comparison of experimental results from Au+Au collisions with collisions of spherical nuclei (e.g. Pb^{207}) at the same energy range and the study of Cu+Cu collisions with the second stable copper isotope at RHIC could be very useful. Elliptic flow studies in collisions of deformed nuclei at LHC would provide us with new and possibly exciting results.

VIII.   APPENDIX: COMPARISON WITH MC GLAUBER CALCULATIONS.

For a comparison with results of full Monte-Carlo Glauber calculation [10] we present here impact parameter dependencies of some quantities obtained with our simple optical Glauber model (OGM) assuming Au^{197} nucleus to be spherical ($R = 6.38$fm; $a = 0.53$).

The impact parameter dependence of the number of binary nucleon-nucleon collisions together with number of participants and spectators calculated in our siplified OGM model

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[3] The influence of $\Delta\phi$ on collision centrality at $b \approx 0$ is rather small (see Fig.8 for Si+Si collisions).
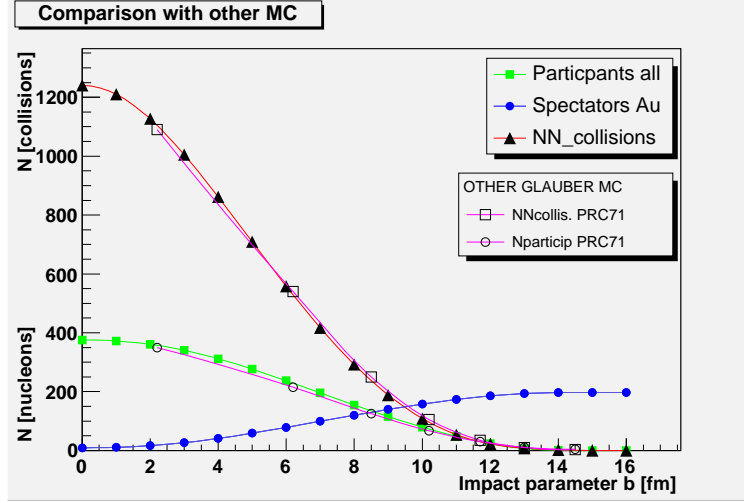
Fig.9: Centrality dependence of binary nucleon-nucleon collisions and participants in Au+Au.

is shown in Figure 9. Data points from full MC Glauber simulation [10] are shown for a comparison.

In Figure 10 we show the maximal transverse baryon density in the overlapping zone and maximal number of nucleon-nucleon collisions per fm^2 in transversal plane for spherical Au collisions. Our simple OGM calculations are in reasonable agreement with results of full MC Glauber simulation [10].
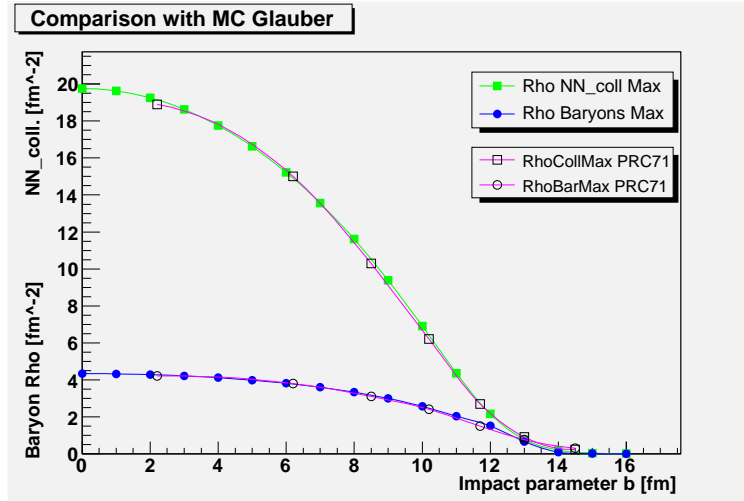

Fig.10: Maximal baryon density $\rho_B^{\mathrm{Max}}[b]$ and maximal collision density $\rho_{\mathrm{N}coll.}^{\mathrm{Max}}[b]$ in Au+Au.

Impact parameter dependence of our calculated (OGM) eccentricity $\tilde{\epsilon}_{NN}$ for Au+Au collisions (assuming spherical Au^{197} nucleus) together with eccentricity values calculated by R.S.Bhalerao et.al. [3] is shown in Figure 11.
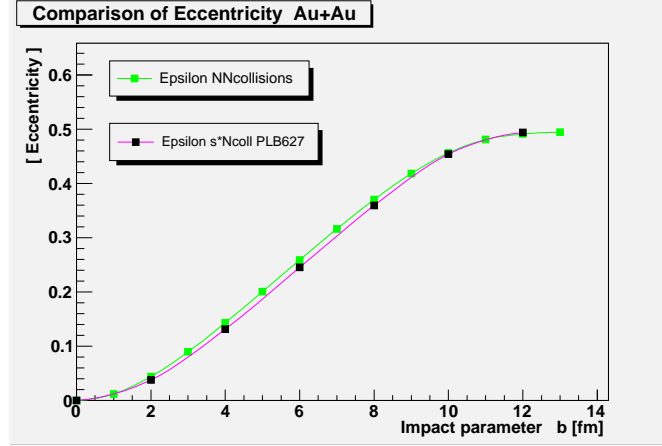
Fig.11: Impact parameter dependence of (OGM) eccentricity $\tilde{\epsilon}_{NN}$ in comparison with results [3].

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